

convergence of distribution expansions. Rates of convergence are also discussed, as are Gibbs phenomena and general partial sums of wavelet expansions. Shannon sampling theorems are presented in Chapter 9, with many different types of sampling.

Chapter 10 is of special interest, as it explores the extent to which translation invariance extends from classical Fourier series/transforms to other orthogonal systems and to what extent there is dilation invariance. The comparison of wavelet, Fourier, and orthogonal polynomial systems, which is one of the central themes of this text, is especially present here. It is shown how specific wavelets can exhibit very simple transformations with respect to translation/dilation, while others have none at all! Weak forms of invariance are also discussed.

In Chapter 11, analytic representations via orthogonal series are discussed, while statistical applications to density estimation and stochastic processes are given in the final two chapters.

Each chapter ends with a problem section: the choice of problems seems quite reasonable and within the range of good graduate students. The bibliography offers students selected references for more detailed study.

This is a most attractive book for mathematicians wishing to learn the basics of wavelet theory, in small doses, and with perspectives given to compare them to more familiar objects. It is also very well suited for its main aim, as a textbook for beginning graduate/senior undergraduate courses on orthogonal systems, harmonic analysis, special functions, and some of their applications.

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Guido Walz, *Asymptotics and Extrapolation*, Mathematical Research **88**, Akademie Verlag, Berlin, 1996, 330 pp.

In numerical analysis and in applied mathematics, many methods produce sequences converging to the solution of the problem. This is the case of iterative methods of various origins and of methods producing a result depending on a parameter (for example, the step size) and tending to the exact solution when the parameter tends to zero. This typical situation arises, for example, in quadrature methods, in discretization methods for ordinary and partial differential equations, and in the approximation of functions.

In many cases, these sequences converge very slowly and it is mandatory to transform them into new sequences converging faster to the same limit. Such transformations are usually called convergence acceleration methods or extrapolation methods. They consist in transforming the slow sequence into another one without modifying the construction of the sequence to be accelerated. The most famous example is Romberg's method for accelerating the trapezoidal rule. Another well-known convergence acceleration method, which works on all linearly converging sequences, is Aitken's \mathcal{A}^2 -process. There exist many extrapolation methods in the literature and the choice of one of them for accelerating the convergence of a given sequence is, of course, based on some theoretical considerations.

It was proved some years ago that, in order to accelerate the convergence of a sequence (x_n) converging to a limit x when n tends to infinity, it is necessary to know an asymptotic expansion of the error $x_n - x$. So, asymptotics and extrapolation methods are two subjects which are, by nature, closely related.

Let us mention that extrapolation methods also lead to new algorithms for the solution of various problems. For example, everyone knows Steffensen's method which converges quadratically for computing fixed points and which is based on Aitken's \mathcal{A}^2 -process. Moreover, convergence acceleration methods are related to many other important topics such as Padé approximation, continued fractions, formal orthogonal polynomials, and projection methods

for systems of linear equations, to name a few. They have many applications, in particular, in physics and chemistry.

The aim of this book is to give an overview of these questions and to show that a profound understanding of asymptotic expansions is necessary to comprehend extrapolation methods seriously. The author has himself contributed significantly to both subjects.

The first part of the book deals with asymptotic expansions: asymptotic systems and expansions, geometric asymptotic expansions, and logarithmic asymptotic expansions. The second part of the book is devoted to linear extrapolation methods: fundamental concepts and general philosophy, error bounds, stopping rules and monotonicity, generalizations, and final remarks. Historical notes and numerical examples have also been included.

This book is well written, well presented, easy to read, and, which is not so common, well printed. It is an important addition to the literature (which is not so abundant) on these topics. It could be used for a course on these questions and it is highly recommended to researchers and to all those who need to use convergence acceleration methods.

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ARTICLE NO. AT973145

Akram Aldroubi and Michael Unser, Eds., *Wavelets in Medicine and Biology*, CRC Press, Boca Raton, FL, 1996, 616 pp.

Wavelets have built a strong reputation in the context of signal and image processing. The editors of this book have invited several specialists to contribute a chapter illustrating this in the (bio)medical and biological sciences.

The book contains four parts. Part I has two chapters written by the editors themselves giving what they call a "surfing guide" to the theory and implementation of the wavelet transform. These 70 pages are among the better introductions to wavelets available in the literature. Both the continuous and the discrete wavelet transform are dealt with, but in view of the applications to follow, the emphasis is on the latter.

The second part deals with medical imaging and tomography. Mathematically speaking, tomography refers to the reconstruction of an image from (noisy) observations or approximations of line integrals which are often called projections. This problem is well known to be ill posed and thus very sensitive to noise. Therefore, a constant observation made in these chapters is that by taking the wavelet transform, it becomes easier to distinguish noise from the clean data. Noise is typically small, of high frequency, and uncorrelated, while the locality of the wavelet basis in both the space and the frequency domains allows one to catch the true image in only a "few" large wavelet coefficients. By a particular way of shrinking the small coefficients, one can "filter out" the noise. Such denoising problems occur in a complex problem setting which depends on the specific application so that several variants and customized versions of this basic idea are explained in these chapters. For example the role of the regularity of the wavelet basis, the basis being orthogonal or biorthogonal, separable or not, the exploitation of redundant transforms versus nonredundant transforms, etc., are all discussed. These denoising techniques are closely related to edge detection and contrast enhancement in images. Indeed, edges correspond to high frequencies, just like noise, but edges give large wavelet coefficients at different resolution levels so that they can be distinguished from noise. The decreasing of small coefficients and the increasing of large ones result in a better contrast of images such as radiographs.

Statistical methods are also an essential tool in these image processing techniques. For example, if the noise level is unknown, statistical techniques are introduced to estimate the noise threshold. Estimation of the local irregularity can reveal whether or not a pixel is